3/27/17

"Yesterday is history, tomorrow a mystery, today is a gift called the present."-Unknown

HW: "Exponential Modeling Day 1" homework section #1-8
Test 3 on Thursday 3/30

AIM: How do we use logs?

Warm Up:

Exponential functions are very important in modeling a variety of real world phenomena because certain things either increase or decrease by **fixed percentages** over given units of time. You looked at this in Common Core Algebra I and in this lesson we will review much of what you saw.

.05

***Exercise* #1:** Suppose that you deposit money into a savings account that receives 5% interest per year on the amount of money that is in the account for that year. Assume that you deposit $400 into the account initially.

(a) How much will the savings account increase by over the course of the year?

$400(.05) = \$20$

(b) How much money is in the account at the end of the year?

$400 + 20 = \$420$

(c) By what single number could you have multiplied the $400 by in order to calculate your answer in part (b)?

$400 + 400(.05) = 420$

$400(1 + .05) = 420$

$400(1.05) = 420$

$\boxed{1.05}$

(d) Using your answer from part (c), determine the amount of money in the account after 2 and 10 years. Round all answers to the nearest cent when needed.

$400(1.05)(1.05)$

$400(1.05)^2 = \$441$

$400(1.05)^{10} = \$651.56$

(e) Give an equation for the amount in the savings account $S(t)$ as a function of the number of years since the $400 was invested.

$S(t) = 400(1 + .05)^t$

$S(t) = 400(1.05)^t$

(f) ~~Using a table on your calculator~~ determine, to the nearest year, how long it will take for the initial investment of $400 to double. Provide evidence to support your answer.

$\$800$

$\frac{800}{400} = \frac{400(1.05)^t}{400}$

$2 = 1.05^t$

$t = \log_{1.05} 2$

$\boxed{t = 14 \text{ years}}$

* Exponential Form

$\text{base}^{\text{exponent}} = \text{answer}$

Ex: $4^2 = 16$

* Log form

$\text{Exponent} = \log_{\text{base}} \text{answer}$

Ex: $2 = \log_4 16$

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* Growth/Decay

$$A(t) = P(1 \pm r)^t$$

(+) for increasing
(-) for decreasing

$A(t)$ = Final Amount

P = Starting Amount

r = rate

t = time

$$A(t) = P(1 \pm r)^t$$

(+) for increasing
(−) for decreasing

$A(t)$ = final amount

P = starting amount

r = rate

t = time

(*) the t and r need to be the same dimension

The thinking process from *Exercise* #1 can be generalized to any situation where a quantity is increased by a fixed percentage over a fixed interval of time. This pattern is summarized below:

INCREASING EXPONENTIAL MODELS

If quantity A is known to increase by a fixed percentage rate r, in decimal form, then A can be modeled by

$$A(t) = P(1+r)^t$$

where P represents the initial amount of present at $t = 0$ and t represents time.

P increasing

***Exercise* #2:** Which of the following gives the savings S in an account if \$250 was invested at an interest rate of 3% per year?

(1) $S = 250(4)^t$ (3) $S = (1.03)^t + 250$

(2) $S = 250(1.03)^t$ (4) $S = 250(1.3)^t$

$r = 3\% = .03$

$S = 250(1+.03)^t$

Decreasing exponentials are developed in the same way, but have the percent subtracted, rather than added, to the base of 100%. Just remember, you are ultimately multiplying by the percent of the original that you will have after the time period elapses.

***Exercise* #3:** State the multiplier (base) you would need to multiply by in order to decrease a quantity by the given percent listed.

(a) 10% (b) 2% (c) 25% (d) 0.5%

(a) $1 - .10 = \boxed{.90}$

(b) $1 - .02$ $\boxed{.98}$

(c) .75

(d) $1 - .005$ $\boxed{.995}$

DECREASING EXPONENTIAL MODELS

If quantity A is known to decrease by a fixed percentage rate r, in decimal form, then A can be modeled by

$$A(t) = P(1-r)^t$$

where P represents the initial amount of present at $t = 0$ and t represents time.

***Exercise* #4:** If the population of a town is decreasing by 4% per year and started with 12,500 residents, which of the following is its projected population in 10 years? Show the exponential model you use to solve this problem.

r P t

(1) 9,230 ~~(3) 18,503~~

(2) 76 (4) 8,310

$A = ?$

$A(t) = 12500(1-.04)^{10}$

$A(10) = 12500(.96)^{10}$

***Exercise* #5:** The stock price of WindpowerInc is increasing at a rate of 4% per week. Its initial value was \$20 per share. On the other hand, the stock price in GerbilEnergy is crashing (losing value) at a rate of 11% per week. If its price was \$120 per share when Windpower was at \$20, after how many weeks will the stock prices be the same? Model both stock prices using exponential functions. Then, find when the stock prices will be equal graphically. Draw a well labeled graph to justify your solution.

r P

Windpower:

$A = 20(1+.04)^t$

$Y_1 = 20(1.04)^x$

Gerbil Energy:

$A = 120(1-.11)^t$

$Y_2 = 120(.89)^x$

$x = 11.5$ weeks